NUMERICAL METHODS FOR PASSIVE COMPONENTS

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ABSTRACT

Because of recent advances in microwave technology, such as monolithic integrated circuits, and the use of the millimetre-wave spectrum, highly sophisticated numerical techniques are required for the analysis and design of microwave circuits and components.

This paper will describe a number of numerical methods which have been developed for the characterization, modelling and design of microwave and millimetre-wave passive structures, with emphasis on planar and quasi-planar structures. Specific advantages and disadvantages in terms of versatility, accuracy, computer time, memory, will be discussed. Typical structures will be illustrated as representative examples of the methods described.

Introduction

The increasing capabilities of computing resources are modifying the way electromagnetic problems are being approached. PC's have computing capabilities comparable with those of the largest computers of a few years ago. On the other hand, main frames and super computers offer enormous computer powers. The advent of extremely powerful and easily accessible computer capabilities has opened new possibilities in the general area of applied electromagnetics and has made it possible to attack a variety of problems that were considered to be inaccessible till a few years ago.

Parallel to the development of computers is the increasing complexity of the electromagnetic structures employed. The use of higher frequency ranges (up to millimeter-waves) and of monolithic circuits involves very complicated circuit configurations. An extremely accurate control is needed of parasitics associated with discontinuities as well as of mechanical tolerances. In addition, the coupling between circuit elements plays an important role in determining the overall circuit performances.

The necessity for accurate numerical tools for the analysis and design of the components and circuits is even more stringent because of the virtual impossibility of tuning the circuits realized.

For the above reasons, numerical methods are an indispensable tool for the analysis and modeling of electromagnetic structures. They form the basis to set up CAD packages.

A number of numerical methods have been proposed and developed since the advent of computers. Each method presents its own advantages and disadvantages. These also depend on the problem under consideration. Selection of the method to be adopted in the solution of a specific structure depends on a number of considerations, such as efficiency, accuracy, memory requirement, versatility. Besides, each method can be implemented according to many different formulations. Different methods lead sometimes to equivalent formulations, as well as they can be combined together into a hybrid formulation. A rigid classification of the various numerical methods is therefore impossible.

This paper provides a quick survey, far from being exhaustive, of the most popular and representative numerical methods for the analysis and characterization of microwave and millimeter-wave passive components. Specific attention is devoted to the most recent circuit configurations of integrated circuits, i.e. planar and quasi-planar structures.

The degree of analytical preprocessing is an important feature of numerical methods. Generally speaking, computer efficiency increases with the amount of analytical effort required but, at the same time, versatility of the method is reduced. This is because analytical forms can be developed for a limited number of simple shapes, or, in any case, under some simplifying hypotheses.

Numerical methods will be presented according, roughly, to the order of increasing analytical complexity and correspondingly decreasing versatility.

1. Finite difference method (FDM) [1-5]

This is the oldest and least analytical method to transform a differential equation into a system of algebraic equations[1-3]. Derivatives are simply replaced by finite differences. The region of interest is divided into nodes located on a 2- or 3- dimensional grid. Because of the very simple algorithm required, the method has a great versatility. It requires, however, a large number of mesh points, thus a large memory storage, and numerical efficiency is rather low. Another shortcoming is the difficulty of fitting curved boundaries with a rectangular mesh. It should be mentioned that FD can be formulated also in conjunction with a variational expression. Because of its versatility, the time domain formulation of the FDM, the so-called TD-FD, is attracting increasing interest among the researchers [4, 5].

2. Transmission Line Matrix (TLM) Method [6-8]

This is similar to the FD-TD method. Actually, it simulates the wave propagation in the time domain by discretizing the space into a 2- or 3-dimensional transmission line matrix. The method is founded on the modeling of the spatial electromagnetic field in terms of a distributed transmission line network. Electric and magnetic fields are made equivalent to voltage and current on the network. The numerical calculation starts by exciting the matrix at specific points by voltage or current pulses. Propagation of pulses is then evaluated at discrete time intervals. Time synchronism is required so that all pulses reach nodes at the same time. Simplicity of formulation and programming, and the calculation of transients are the main attractives of the method.

3. Finite Element Method (FEM) [9-14]

This method is implemented on an integral (variational) formulation of the boundary value problem. The region of interest is subdivided into surface or volume elements where the unknown function is approximated as a polynomial. Rayleigh-Ritz procedure transforms the functional minimization into a linear system of algebraic equations.

FEM has certain advantages of flexibility over FDM, as odd shaped boundaries can be fitted easily. Boundary conditions can be incorporated into the variational expression. The order of the polynomial approximation is an additional degree of freedom for numerical computations. Care must be excercised because of the possible existence of spurious (non physical) solutions.

4. Boundary element method (BEM) [15-17]

The wave equation is converted into an integral over the boundary by way of Green's identity. A discretization procedure similar to the FEM is then applied. Over FEM, this method has the advantage of the reduction of one dimension, thus of CPU time and memory storage. BEM for planar structures has been recently formulated without the use of Green's function [17].

5. Method of Moments (MOM) [18-19]

This is a very general approach to convert an analytical formulation of a boundary value problem into a numerical formulation in the form of a linear system of algebraic equations. The analytical formulation may be in the form of a differential or integral equation that can be put in the form

$$L f = g$$

where L is a linear operator, f is the unknown function and g is the excitation. The unknown f is expanded in terms of basis functions f_n and the resulting equation is weighted by a set of testing functions w_m. This is made in terms of a suitably defined inner product. This procedure results in a linear matrix equation in the expansion coefficients.

MOM is often used in conjunction with an integral equation formulation.

A number of numerical methods derive from the above general scheme. They differ from the choice of basis and testing functions. When these are made identical Galerkin's method is obtained. This method is known to be equivalent to Rayleigh-Ritz variational procedure. Finite element method, mode matching method and others can also be regarded as special cases of MOM.

The numerical solution of the final matrix equation is an essential step towards the field solution. This is not a trivial problem, particularly for large systems. The conjugate gradient method (CGM) is probably the most popular method for solving matrix equations resulting from the application of the MOM [20-21]. The CGM can be incorporated into the MOM itself so as to adaptively modify the basis and testing functions.

6. Mode Matching Technique (MMT)[22-24]

This is a classical method for solving waveguide discontinuity problems. The fields on both sides of the discontinuity are expanded in terms of normal modes of the respective guides. By virtue of orthogonal properties, the boundary conditions are transformed into a matrix equation in the expansion coefficients. A quite similar technique, which strictly speaking should be referred to as field-matching, can be used to compute the normal modes of waveguides with complicated cross sectional geometries [25].

The modal decomposition of the field at discontinuities, which forms the basis of MMT, is also the basis of a number of computational schemes for the characterization of cascaded discontinuities. [26-28]

7. Transverse Resonance Technique (TRT) [29-33]

This technique was introduced many years ago as an application of the microwave network formalism in the analysis of waveguides and leaky-wave antennas [29-30]. The modal network formalism is applied in a direction perpendicular to the actual power flow. Propagation characteristics of waveguides containing longitudinal discontinuities, such as metallic fins, grooves, ridges, etc., are computed in a straigthforward manner from the transverse resonance condition. This technique requires the knowledge of an accurate equivalent circuit representation of the discontinuity.

TRT has been recently generalized so as to provide full wave analyses of both uniform [31-32] and discontinuous [33] printed circuit transmission lines. The method consists basically of a MMT applied in the direction perpendicular to the plane of the circuit.

8. Spectral Domain Method (SDM) [34-36]

This is probably the most popular method for the analysis of planar and quasi-planar structures, such as microstrip, coplanar lines, finline, etc. Field components are Fourier transformed in both x and z directions of the plane of the circuit (y=0). The boundary conditions at y=0 result in two algebraic equations relating the tangential E-field components to the current density components over the metallization. This matrix relation involves the Fourier transform of the diadic Green's function. Galerkin's method is then applied expanding either the electric field or the current density in a suitable set of basis functions. Choice depends on the prevalence of the metallized (e.g. slotline) or non-metallized (e.g. microstrip) portion of the interface y=0. The derivation of the transformed Green's function can be significantly simplified by the generalized immittance approach[37].

SDM requires a considerable amount of analytical preprocessing, but, by a proper choice of the basis functions it leads to very small matrix sizes. Efficiency is high, but applicability is limited to well-shaped structures.

9. Method of lines (ML) [38, 39]

This is also a method for analyzing planar structures of arbitrary geometry. It is a hybrid method since it combines FDM with an analytical treatment of the field equations. These are discretized in the plane of the circuit. The resulting matrix differential equation can be diagonalized by a suitable orthogonal transformation so that, using a transmission line representation normal to this plane, a matrix equation is obtained in the transformed domain. This matrix equation relates the tangential E-field components to the current density over the metallization. The vanishing of the tangential electric field finally result in an eigenvalue equation in the propagation constant or in the resonant frequency.

10. Planar circuit approach (PCA) [15, 40-44]

The concept of planar circuit was introduced by Okoshi and Miyoshi [15], as a circuit element in which one dimension is much smaller than the wavelength. More than a numerical method, the PCA may be regarded as a general mathematical formalism to analyze planar structures. The planar waveguide model (PWM) is used to characterize microstrip lines and discontinuities by transforming the open microstrip discontinuity into a closed waveguide discontinuity problem [40]. Planar circuit models may be considered as an extension of the PWM [41]. A rigorous mathematical formalism in terms of two-dimensional telegraphists' equations can be derived [42]. In the frame of the planar approach, specific numerical techniques, such as segmentation and desegmentation methods [43,44] have been developed to broaden the applicability to complicated circuit configurations.

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